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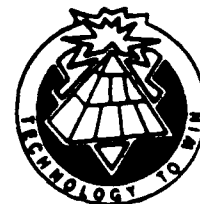
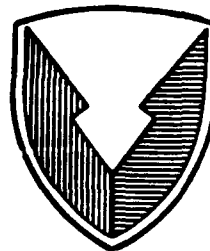
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ESTIMATING  $C_n^2$ ,  $C_t^2$ ,  $C_{tq}$ , AND  $C_q^2$   
DURING UNSTABLE ATMOSPHERIC CONDITIONS

August 1991

Henry Rachele  
Arnold Tunick



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US ARMY  
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## 1. INTRODUCTION

Andreas (1988) in referencing Wyngaard et al. (1971) states that the structure parameter  $C_t^2$  should have a universal form when properly scaled, that is,

$$\frac{C_t^2 z^{2/3}}{t_*^2} = g_t \left( \frac{z}{L} \right) \quad (1)$$

Where, for unstable conditions,

$$g_t \left( \frac{z}{L} \right) = 4.9 \left[ 1 - 6.1 \frac{z}{L} \right]^{-2/3} \quad (2)$$

$z$  - height

$t_*$  - temperature scaling length

$L$  - Obkuhov (1946) length

Andreas also notes that Fairall et al. (1980a) and Kohsiek (1982) extended these arguments to the humidity structure parameter  $C_Q^2$  and to the temperature-humidity structure parameter  $C_{tQ}$  obtaining

$$\frac{C_Q^2 z^{2/3}}{Q_*^2} = g_q \left( \frac{z}{L} \right) \quad (3)$$

and

$$\frac{C_{tQ} z^{2/3}}{t_* Q_*} = g_{tq} \left( \frac{z}{L} \right) \quad (4)$$

where,

$Q$  - absolute humidity (grams per cubic meter)

$Q_*$  - absolute humidity scaling length

Andreas further suggests that  $C_n^2$  should exhibit the characteristics of  $C_t^2$ ,  $C_Q^2$ , and  $C_{tQ}$  since it can be shown that

$$C_n^2 = A^2 C_t^2 + 2 A B C_{tQ} + B^2 C_Q^2 \quad (5)$$

where A and B are derived from a real index of refraction fluctuation expression that varies with wavelength. Consistent with the above formulations and concepts, Andreas developed a similarity expression for the real index of refraction fluctuations,

$$n_s = A t_s + B Q_s \quad (6)$$

and concludes that  $C_n^2$  should have the form

$$\frac{C_n^2 z^{2/3}}{n_s^2} = g_n \left( \frac{z}{L} \right) \quad (7)$$

Later in his report, Andreas (1988) concludes and Hill (1989) proves that

$$g_t \left( \frac{z}{L} \right) = g_q \left( \frac{z}{L} \right) = g_{tq} \left( \frac{z}{L} \right) = g_n \left( \frac{z}{L} \right) = g \left( \frac{z}{L} \right) \quad (8)$$

We wish to emphasize that up to this point in this report that Q represents moisture in absolute humidity form; whereas, in later developments we will use q (specific humidity) to be constant with Tatarski's (1961) original derivations of  $C_n^2$ .

The purpose of this report is to present our development of expressions for  $C_t^2$ ,  $C_{tq}$ ,  $C_q^2$ ,  $C_n^2$ ,  $g_t \left( \frac{z}{L} \right)$ ,  $g_q \left( \frac{z}{L} \right)$ ,  $g_{tq} \left( \frac{z}{L} \right)$ , and  $g_n \left( \frac{z}{L} \right)$ ; comparisons with those in Andreas; and comparisons of  $C_n^2$  with Wesely (1976).

## 2. RACHELE/TUNICK (RT) FORMULATIONS

We initiate our development using Tatarski's expression for  $C_n^2$ , that is,

$$C_n^2 = b \left( \frac{K_B}{\epsilon^{1/3}} \right) \left( \frac{dn}{dz} \right)^2 \quad (9)$$

where,

b = constant = 3.2 (Wyngaard (1973), Hill (1989), Andreas (1988))

$$K_B = \frac{u' k z}{\phi_B} = \text{turbulent exchange coefficient for heat} \quad (10)$$

$$\epsilon = \frac{u^*{}^3}{kz} \left( \phi_M - \frac{z}{L} \right) = \text{energy dissipation rate} \quad (11)$$

$$\phi_B = \left( 1 - \gamma \frac{z}{L} \right)^{-1/2}, \quad L < 0 \quad (12)$$

$$\phi_M = \left( 1 - \beta \frac{z}{L} \right)^{-1/4}, \quad L < 0 \quad (13)$$

$k$  = Karman's constant (0.4)

$u^*$  = friction velocity

$\gamma = \beta = 15$  (Hansen, 1980)

$\frac{dn}{dz}$  = the gradient of the real index of refraction in the vertical direction  $z$ .

Andreas' (1988) expression for  $n$  with moisture in absolute humidity is

$$n = A(\lambda, P, T, Q) T + B(\lambda, P, T, Q) Q, \quad (14)$$

where for  $\lambda = 0.55 \mu m$

$$A = -7.9 \times 10^{-5} \frac{P}{T^2}$$

$$B = -5.46 \times 10^{-5}$$

and where  $P$  and  $T$  are pressure in millibars and temperature in K.

In our notation we write equation (14) as

$$dn = A(\lambda, P, T, q) dT + B_q(\lambda, P, T, q) dq \quad (15)$$

where,



$$A = - 7.9 \times 10^{-5} \frac{P}{T^2}$$

$$B_q = - 1.97 \times 10^{-5} \frac{P}{T}$$

q = specific humidity

Dividing equation (15) by dz gives

$$\frac{dn}{dz} = - 7.9 \times 10^{-5} \frac{P}{T^2} \frac{dT}{dz} - 1.97 \times 10^{-5} \frac{P}{T} \frac{dq}{dz} \quad (16)$$

Furthermore, equation (5), in terms of specific humidity, becomes

$$C_n^2 = A^2 C_t^2 + 2 A B C_{tq} + B_q^2 C_q^2. \quad (17)$$

For unstable homogeneous conditions, we let

$$\frac{dT}{dz} = \frac{\partial \theta}{\partial z} \text{ and } \frac{dq}{dz} = \frac{\partial q}{\partial z} \quad (18)$$

Substituting equation (18) into equation (16) gives

$$\frac{dn}{dz} = - 7.9 \times 10^{-5} \frac{P}{T^2} \frac{\partial \theta}{\partial z} + - 1.97 \times 10^{-5} \frac{P}{T} \frac{\partial q}{\partial z} \quad (19)$$

or

$$\frac{dn}{dz} = - 7.9 \times 10^{-5} \frac{P}{T^2} \frac{\partial \theta}{\partial z} - (1.975 \times 10^{-5} T) \frac{P}{T^2} \frac{\partial q}{\partial z} \quad (20)$$

Squaring equation (20) gives

$$\begin{aligned}
\left(\frac{dn}{dz}\right)^2 &= 6.241 \times 10^{-9} \frac{P^2}{T^4} \left(\frac{\partial \theta}{\partial z}\right)^2 \\
&+ [3.11 \times 10^{-9} T] \frac{P^2}{T^4} \cdot \frac{\partial \theta}{\partial z} \frac{\partial q}{\partial z} \\
&+ [3.88 \times 10^{-10} T^2] \frac{P^2}{T^4} \left(\frac{\partial q}{\partial z}\right)^2
\end{aligned} \tag{21}$$

Expressing  $\frac{\partial \theta}{\partial z}$  and  $\frac{\partial q}{\partial z}$  in equation (21) in similarity form, for unstable conditions, that is,

$$\frac{\partial \theta}{\partial z} = \frac{\theta^*}{kz} \left[1 - \gamma \frac{z}{L}\right]^{-1/2} \tag{22}$$

equation (21) becomes

$$\frac{\partial q}{\partial z} = \frac{q^*}{kz} \left[1 - \gamma \frac{z}{L}\right]^{-1/2} \tag{23}$$

$$\begin{aligned}
\left(\frac{dn}{dz}\right)^2 &= 6.241 \times 10^{-9} \frac{P^2}{T^4} \frac{\theta^{*2}}{k^2 z^2} \left[1 - \gamma \frac{z}{L}\right]^{-1} \\
&+ 3.11 \times 10^{-9} T \frac{P^2}{T^4} \frac{\theta^* q^*}{k^2 z^2} \left[1 - \gamma \frac{z}{L}\right]^{-1} \\
&+ 3.88 \times 10^{-10} T^2 \frac{P^2}{T^4} \frac{q^{*2}}{k^2 z^2} \left[1 - \gamma \frac{z}{L}\right]^{-1}
\end{aligned} \tag{24}$$

where

$\gamma$  = constant

$$L = \frac{u^{*2} T (1 + 0.61 q)}{kg (\theta^* + 0.61 \theta q^*)} \tag{25}$$

$g$  = acceleration of gravity.

Substituting equation (24) into equation (5) and (9) gives

$$C_n^2 = A' \theta^{*2} + B' \theta^* q^* + C' q^{*2} \quad (26)$$

where

$$A' = b \left( \frac{K_H}{\epsilon^{1/3}} \right) 6.241 \times 10^{-9} \frac{P^2}{T^4 k^2 z^2} \left( 1 - \gamma \frac{z}{L} \right)^{-1} \quad (27)$$

$$B' = b \left( \frac{K_H}{\epsilon^{1/3}} \right) (3.11 \times 10^{-9} T) \frac{P^2}{T^4 k^2 z^2} \left( 1 - \gamma \frac{z}{L} \right)^{-1} \quad (28)$$

$$C' = b \left( \frac{K_H}{\epsilon^{1/3}} \right) (3.88 \times 10^{-10} T^2) \frac{P^2}{T^4 k^2 z^2} \left( 1 - \gamma \frac{z}{L} \right)^{-1} \quad (29)$$

$$K_H = \frac{k u^* z}{\left( 1 - \gamma \frac{z}{L} \right)^{-1/2}} \quad (30)$$

$$\epsilon = \frac{u^{*3}}{kz} \left( \phi_M - \frac{z}{L} \right) \quad (31)$$

$$\phi_M = \left( 1 - \beta \frac{z}{L} \right)^{-1/4} \quad (32)$$

From equations (30) through (32)

$$\frac{K_H}{\epsilon^{1/3}} = k^{4/3} z^{4/3} \left\{ \frac{\left( 1 - \gamma \frac{z}{L} \right)^{1/2}}{\left[ \left( 1 - \beta \frac{z}{L} \right)^{-1/4} - \frac{z}{L} \right]^{1/3}} \right\} \quad (33)$$

Substituting equations (33) into equations (27), (28), and (29) gives

$$A' = b \left[ 6.241 \times 10^{-9} \frac{P^2}{T^4} \right] k^{-2/3} z^{-2/3} \left[ 1 - \gamma \frac{z}{L} \right]^{-1} \left\{ \right\} \quad (34)$$

$$B' = b \left[ 3.11 \times 10^{-9} T \right] \cdot \frac{P^2}{T^4} k^{-2/3} z^{-2/3} \left[ 1 - \gamma \frac{z}{L} \right]^{-1} \left\{ \right\} \quad (35)$$

$$C' = b \left( 3.88 \times 10^{-10} T^2 \right) \frac{P^2}{T^4} k^{-2/3} z^{-2/3} \left[ 1 - \gamma \frac{z}{L} \right]^{-1} \left\{ \right\} \quad (36)$$

Equating equations (26) and (17) term by term gives

$$C_t^2 = \frac{A'}{A^2} \theta^{*2} \quad (37)$$

$$= b \theta^{*2} k^{-2/3} z^{-2/3} \left[ 1 - \gamma \frac{z}{L} \right]^{-1} \left\{ \right\} \quad (38)$$

$$= b \theta^{*2} k^{-2/3} z^{-2/3} \left\{ \frac{\left[ 1 - \gamma \frac{z}{L} \right]^{-1/2}}{\left[ \left[ 1 - \beta \frac{z}{L} \right]^{-1/4} - \frac{z}{L} \right]^{1/3}} \right\} \quad (39)$$

or

$$\frac{C_t^2 z^{2/3}}{\theta^{*2}} = b k^{-2/3} \left\{ \frac{\left[ 1 - \gamma \frac{z}{L} \right]^{-1/2}}{\left[ \left[ 1 - \beta \frac{z}{L} \right]^{-1/4} - \frac{z}{L} \right]^{1/3}} \right\} = g_t \quad (40)$$

Similarly,

$$C_{tq} = \frac{B' \theta^* q^*}{2 A B_q} \quad (41)$$

$$\dot{=} b \theta^* q^* k^{-2/3} \left\{ \right\} = g_{tq} \quad (42)$$

or

$$\frac{C_{tq} z^{2/3}}{\theta^* q^*} \dot{=} b k^{-2/3} \left\{ \right\} = g_{tq} \quad (43)$$

Also

$$C_q^2 = \frac{C' q^{*2}}{B_q^2}$$

$$\dot{=} b q^{*2} k^{-2/3} z^{-2/3} \left\{ \right\} \quad (44)$$

or

$$\frac{C_q^2 z^{2/3}}{q^{*2}} = b k^{-2/3} \left\{ \right\} = g_q \quad (45)$$

To account for moisture contributions to  $C_n^2$ , Kunkel and Walters (1983) used an expression by Wesely (1976), that is,

$$C_n^2 = C_t^2 A^2 (1 + 0.03/B_o)^2 \quad (46)$$

where

$$B_o = \text{Bowen ratio} = 4 \times 10^{-4} \frac{\theta^*}{q^*} \quad (47)$$

In our model we relate  $C_n^2$  to  $C_t^2$  and  $B_o$  as follows. From equations (37) through (45) we write

$$C_t^2 = \theta^{*2} z^{-2/3} g_t \quad (48)$$

$$C_{tq} = \theta^* q^* z^{-2/3} g_{tq} = \theta^* q^* z^{-2/3} g_t \quad (49)$$

$$C_q^2 = q^{*2} z^{-2/3} g_q = q^{*2} z^{-2/3} g_t \quad (50)$$

Furthermore, from equations (48) through (50), we obtain

$$C_{tq} = \frac{q^*}{\theta^*} C_t^2 \frac{g_{tq}}{g_t} = \frac{q^*}{\theta^*} C_t^2 \quad (51)$$

$$C_q^2 = \frac{q^{*2}}{\theta^{*2}} \frac{g_q}{g_t} C_t^2 = \left( \frac{q^*}{\theta^*} \right)^2 C_t^2 \quad (52)$$

Substituting equations (51) and (52) into equation (17) gives

$$C_n^2 = A^2 C_t^2 \left[ 1 + \frac{B_q}{A} \frac{q^*}{\theta^*} + \left( \frac{B_q}{A} \right)^2 \left( \frac{q^*}{\theta^*} \right)^2 \right] \quad (53)$$

or, in terms of  $B_o$ ,

$$C_n^2 = A^2 C_t^2 \left[ 1 + \frac{B_q}{AB_o} + \frac{1.6 \times 10^{-7}}{B_o^2} \left( \frac{B_q}{A} \right)^2 \right] \quad (54)$$

Substituting the values of  $A$  and  $B_q$ , given earlier (equation (15)), into equation (54) results in

$$C_n^2 = A^2 C_t^2 \left[ 1 + \frac{1.99 \times 10^{-4}}{B_o} T + \frac{3.98 \times 10^{-8}}{B_o^2} T^2 \right] \quad (55)$$

### 3. COMPARISONS

Wyngaard (1973) concluded from scaling laws and the analyses of Kansas data that

$$g_t(\zeta) = 4.9 (1 - 6.1\zeta)^{-2/3} \quad (56)$$

for  $\zeta \leq 0$ . Hill (1989) demonstrated that  $g_t(\zeta) = g_{tq}(\zeta) = g_q(\zeta) = g_n(\zeta)$ .

Andreas (1988) concluded that

$$g_b(\zeta) = \frac{5.92\phi_B(\zeta)}{\phi_e(\zeta)^{1/3}} \quad (57)$$

based on the budget equations where Wyngaard and Cote (1971), Kaimal et al. (1972), Fairall et al. (1980b), and Schacher et al. (1981) found that

$$\phi_e(\zeta) = [1 + 0.46(-\zeta)^{2/3}]^{3/2}, \quad 2 \leq \zeta \leq 0 \quad (58)$$

$$\phi_e(\zeta) = [1 + 2.3 \zeta^{3/5}]^{3/2}, \quad 0 \leq \zeta \leq 2 \quad (59)$$

From the formulations in this study (equation (40)), we found that

$$g_t(\zeta) = \frac{5.89\phi_B(\zeta)}{\phi_e(\zeta)^{1/3}} \quad (60)$$

where, from Panofsky (1968),  $\phi_e(\zeta)^{1/3} = (\phi_m - \zeta)^{1/3}$ .

We also show that  $g_t(\zeta) = g_q(\zeta) = g_{tq}(\zeta) = g_n(\zeta)$ .

The functions  $(1 - 6.1\zeta)^{-2/3}$ ,  $\frac{\phi_B(\zeta)}{[1 + 0.46(-\zeta)^{2/3}]^{3/2}}$  and  $\frac{\phi_B(\zeta)}{(\phi_m - \zeta)^{1/3}}$  are compared in

figure 1; whereas, the complete functions

$4.9(1 - 6.1\zeta)^{-2/3}$ ,  $\frac{5.92\phi_B(\zeta)}{[1 + 0.46(-\zeta)^{2/3}]^{3/2}}$ , and  $\frac{5.89\phi_B(\zeta)}{(\phi_m - \zeta)^{1/3}}$  are shown in figure 2.

Figure 3 illustrates resultant  $C_n^2$  vertical profiles by using the three forms for  $g(z/L)$  discussed above. Equations (6) and (7) describe the formulation for the profiles presented in figure 3.

Finally, we compare values of  $C_n^2/C_t^2$  by using equations (46) and (55). These values are presented in figure 4.

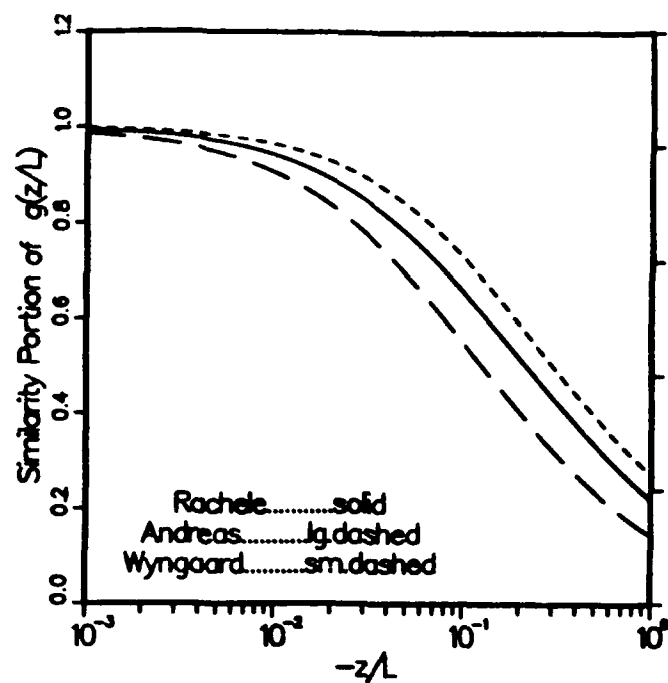


Figure 1. A comparison of the variations of the similarity portion of the function  $g(z/L)$ .

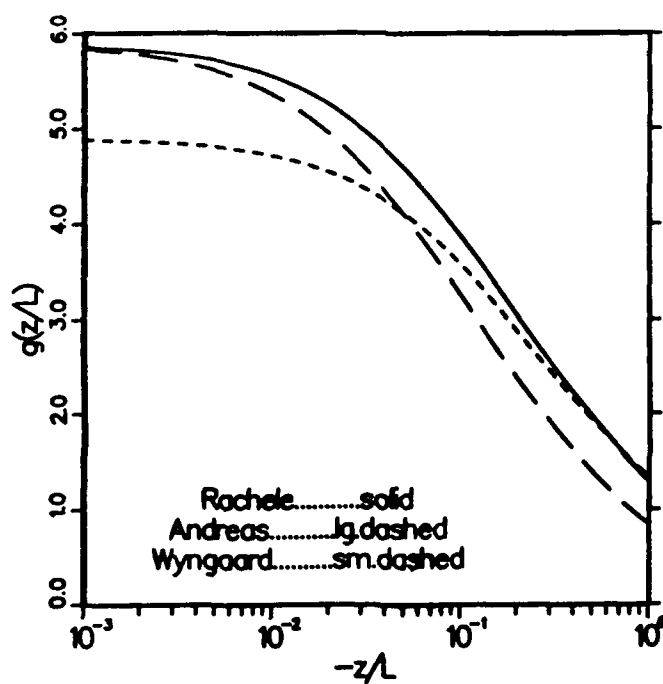


Figure 2. A comparison of the complete expressions of the function  $g(z/L)$ .



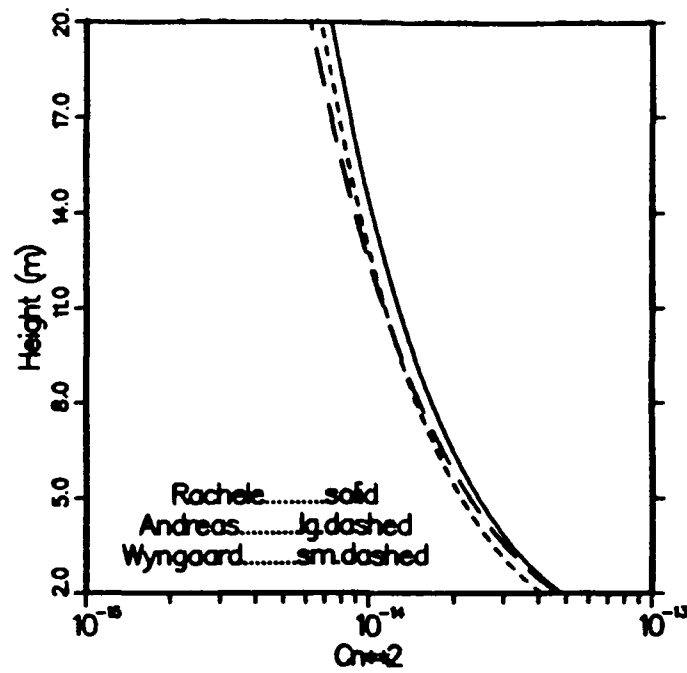


Figure 3.  $C_n^2$  vertical profiles using the  $g(z/L)$  functions of Rachele, Andreas, and Wyngaard.

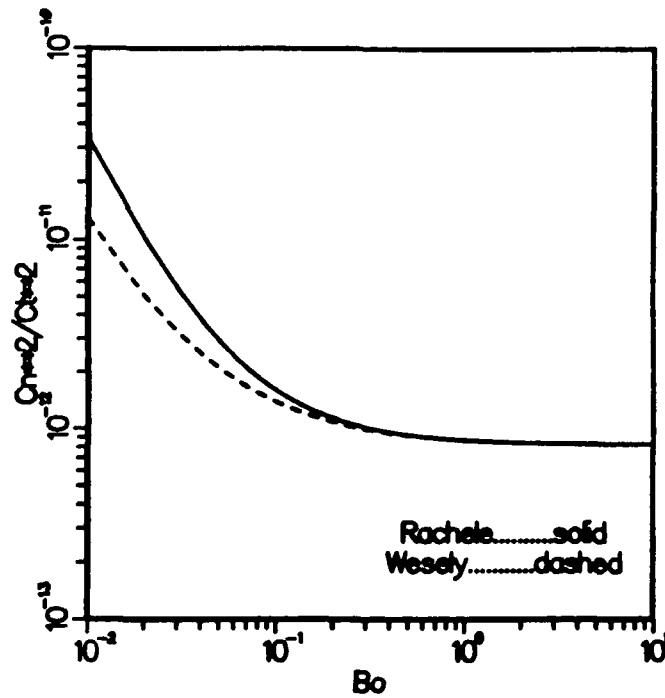


Figure 4. The ratio  $C_n^2/C_t^2$  versus  $B_o$  (Bowen Ratio) for equations (46) (Wesely, 1976) and (55) (Rachele) for  $T = 296.15$  K.

#### 4. SUMMARY AND CONCLUSIONS

In this report, we have shown our development of expressions for the refractive index structure parameters,  $C_n^2$ , as well as the temperature and humidity structure function parameters  $C_t^2$ ,  $C_q^2$ , and  $C_{tq}$ . We have also presented our development for the similarity functions  $g_t$ ,  $g_{tq}$ , and  $g_q$ .

We concluded that the singular universal function  $g(z/L)$  satisfies the expression  $g_t = g_{tq} = g_q = g$  as shown by equations (40), (43), and (45).

In figures 1 and 2 we show that as  $|-z/L|$  increases in magnitude greater differences arose between the expressions for  $g_t$  of Andreas (1988), Wyngaard (1973), and Rachele. Clearly they are not equivalent.

However, much less separation is evidenced between the profiles of  $C_n^2$  presented in figure 3. The choice of which  $g(z/L)$  function to use may now be less critical.

Finally, we compared values of the ratio  $C_n^2/C_t^2$  by using equations from our development and those from Wesely (1976). Figure 4 shows that both expressions compare closely to one another.

# LITERATURE CITED

- Andreas, Edgar L., 1988, "Estimating  $C_n^2$  Over Snow and Sea Ice from Meteorological Data," J Opt Soc Am, 5:481-495.
- Fairall, C. W., G. E. Schacher, and K. L. Davidson, 1980a, "Measurements of the Humidity Structure Function Parameters,  $C_q^2$ , and  $C_{tq}$ , over the Ocean," Boundary Layer Meteorol, 19:81-92.
- Fairall, C. W., R. Markson, G. E. Schacher, and K. L. Davidson, 1980b, "An Aircraft Study of Turbulence Dissipation Rate and Temperature Structure Function in the Unstable Marine Atmospheric Boundary Layer," Boundary Layer Meteorol, 19:453-469.
- Hansen, F., 1980, Flux Profile Relationships for Development of Standards of Comparison, ASL Internal Report, U.S. Army Atmospheric Sciences Laboratory, White Sands Missile Range, NM.
- Hill, R. J., 1989, "Implications of Monin-Obukhov Similarity Theory for Scalar Quantities," J Atmos Sci, 46:2236-2244.
- Kaimal, J. C., J. C. Wyngaard, Y. Izumi, and O. R. Cote, 1972, "Spectral Characteristics of Surface-Layer Turbulence," Quart J Roy Meteorol Soc, 98:563-589.
- Kohsiek, W., 1982, "Measuring  $C_t^2$ ,  $C_q^2$ , and  $C_{tq}$  in the Unstable Surface Layer, and Relations to the Vertical Fluxes of Heat and Moisture," Boundary Layer Meteorol, 24:89-107.
- Kunkel, K. E., and D. L. Walters, 1983, "Modeling the Diurnal Dependence of the Optical Refractive Index Structure Parameter," J Geophys Res, 88:10999-11004.
- Obukhov, A. M., 1946, "Turbulence in an Atmosphere of Nonhomogeneous Temperature," Trans Inst Theort Geophys, USSR, 1:95.
- Panofsky, H. A., 1968, "The Structure Constant for the Index Refraction in Relation to the Gradient of Index of Refraction in the Surface Layer," J Geophys Res, 73(18):6047-6049.
- Schacher, G. E., K. L. Davidson, T. Houlihan, and C. W. Fairall, 1981, "Measurements of the Rate of Dissipation of Turbulence Kinetic Energy,  $\epsilon$ , over the Ocean," Boundary Layer Meteorol, 20:321-330.
- Tatarski, V. I., 1961, Wave Propagation in a Turbulent Medium, McGraw-Hill Book Company, Inc., New York - Toronto - London.
- Wesely, M. L., 1976, "The Combined Effect of Temperature and Humidity Fluctuations on Refractive Index," J Appl Meteorol, 15:43-49.

Wyngaard, J. C., Y. Izumi, and S. A. Collins, Jr., 1971, "Behavior of the Refractive-Index-Structure Parameter Near the Ground," J Opt Soc Am, 61:1646-1650.

Wyngaard, J. C., and O. R. Cote, (1971), "The Budgets of Turbulent Kinetic Energy and Temperature Variance in the Atmospheric Surface Layer," J Atmos Sci, 28:190-201.

Wyngaard, J. C., 1973, "On Surface-Layer Turbulence," in Workshop on Micrometeorology, D. A. Haugen, editor, American Meteorological Society, Boston, MA, 101-149.

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